On the use of long–term risk measures

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Introduction

- Value at risk (VaR) and its modifications, such as TVaR are widely used risk measures.
- Let $\{L(t)\}_{t\geq 0}$ denote a stochastic process representing the losses of a portfolio over time,
- The VaR with time horizon t and probability level α is defined as

$$VaR_\alpha(L(t)) = \inf\{l, \mathbb{P}(L(t) > l) \leq 1-\alpha\} = \inf\{l, \mathbb{P}(U(t) \leq l) \geq \alpha\}.$$

- The Tail Value at risk (TVaR) is defined by

$$TVaR_\alpha(L(t)) = \mathbb{E}[L(t)|L(t) > VaR_\alpha(L(t))].$$

Introduction

- In banking, some commonly used parameter values for VaR are 1% and 5% probabilities and one day and two week horizons. In insurance regulation, the Solvency Capital Requirement (SCR) of solvency II requires a capital level corresponding to one year VaR at level 99.5%.

Introduction

- (Einhorn 2008) argues that VaR ignores what happens in the tails. A 99% VaR calculation does not evaluate what happens in the last 1%. Consequently, VaR creates an incentive to take excessive but remote risks.
- One could consider the mean of the losses that exceeds the VaR level, by considering TVaR as proposed in Artzner (1999). However, Wang (2002) argued that TVaR does not adequately adjust for extreme low-frequency and high-severity losses, since it only accounts for the mean shortfall (not higher moments).

Introduction

- Taleb (2012), "VaR encourages low volatility, high blowup risk taking which can be gamed by the Wall Street bonus structure" It was also argued that one reason for this is the limitation of ability of all quantitative risk measures (including VaR, TVaR and many other modifications) to measure the risk of extreme events (black swans).
- This was evidenced by that fact that many traders sell out-of-the-money options, with the payoff stream being frequent (small) profits and infrequent large losses (Taleb, 2012).

Introduction

- In addition to the problem of ignoring tail, Taleb (2004, 2012) argued that VaR (as well as TVaR) has the side effect of "anchoring" (Tversky and Kahneman 1974) in decision making. That is, because VaR indicates a small probability of losses of certain size within a short time-horizon, it communicates a false feeling of safety to the decision makers.

Introduction

- It is known that under certain situation, people intend NOT to protect themselves against losses whose probability is below some threshold.
- For example, Slovic et. al. (1977) found that people buy more insurance against events having a moderately high probability of inflicting a relatively small loss than against low probability, high–loss events.
- Slovic et. al.(1978) concluded that many people do not wear seat belts because they believe that the chance of death or injury from a car accident is too low to be of concern–according to Wikipedia, the road fatalities per 1–billion vehicle–km is 8.5 in the United States in 2009.

Introduction

- Notice that financial instrument traders or insurance companies have even less incentives to manage extreme events with small occurrence probability because investors (policy holders) and/or taxpayers eventually pay the bill when a catastrophe does occur.

Introduction

- To remedy the problem of dealing with small probability, Slovic (1977) suggested that one can (1) combine low probability hazards with higher probability threats in one insurance "package" (see also Kunreuther and Pauly (2004)); and (2) compound the hazard over time.

Introduction

- Through experiments, Slovic et. al. (1977) concluded that "it does appear possible that multiple exposures can induce people to purchase insurance by boosting the overall probability of loss".

Introduction

- In an experiment reported in Slovic et. al.(1978), one group of people were informed that the car accident disabling injury rate is one per 100,000 trips; another groups of people was informed that over a lifetime of driving (assumed to be 40000), the probability of a disabling injury is 0.33. It was found the 39% of the respondes who were given the lifetime probability said they would use seat belt, whereas 10% of those provided with single–trip probability said so.

Introduction

- In an investment setting, VaR indicates a low probability of incurring a loss of certain size in a short time horizon. However, when viewed from a long–term perspective, these low probabilities can cumulated to a (both mentally and physically) substantial level.
- For example, if a fund manager maintain a daily VaR of \$1 million at a 1% probability, then the chance of incurring a \$1 million loss over a year (assuming 250 trading days) is $1 - 0.99^{250} = 92\%$!

Introduction

- Duffie and Pan (1997) pointed out that " given the overriding goal of protecting the franchise value of the firm, one should not treat one's measure of Value at Risk, even if accurate, as the level of capital necessary to sustain the firm's risk. Value at Risk is merely a benchmark for relative judgements such as the risk of one desk relative to another, the risk of one portfolio relative to another, $\cdots$ "

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Introduction

- Here, analogue to the results in Slovic et. al.(1978) for seat belt usage, we hypothesize that explicitly communicating the long–term cumulative probability corresponding to the one period VaR value might help alleviating the anchoring problem introduced by the small probability values associated with VaR.
- That way, investors are less likely to tolerate trading strategy that create chances of blowup with probability so low that are not detected by VaR or insensitive to human minds.

Introduction

- Of course, this argument needs to be tested empirically, which should be performed in the future (somehow).
- Instead, we focus on the modelling of the accumulation of risk and ways to measure the accumulated risks.

Introduction

- When cumulating short–term risk temporally in a repeating Bernoulli–trial fashion, the benefit of bearing the risk (the risk premiums earned) is not considered. For example, one might prefer a position (A) with 0.1% daily VaR of $10 million to a position (B) with 1% daily VaR of $1 million not only because A has lower probability of occurring, but also because of the possible risk premiums earned before it actually occurs.

Introduction

- Risk processes developed in actuarial risk literature (see for example, Bowers et al. 1997) may be applied to model the accumulation of risk and the risk premiums.

Introduction

- A risk process is typically defined by

$$U(t) = u + c(t) - S(t),$$

where u is the insurer's initial capital level, $c(t)$ is the premium income collected, and $S(t)$ is the cumulative losses during time interval $(0, t]$.
- The ruin probability of the process $U(t)$ with time horizon T is defined by
-

$$\psi(u, T) = \mathbb{P}\left(\inf_{0<t\leq T}\{U(t)\} < 0\right). \tag{1}$$

Example 1

- Consider underwriting two potential risks X and Y, where X may cause an annual loss of \$600 with probability $p = 0.001$ and zero otherwise, Y may cause an annual loss of \$300 with probability $p = 0.002$ and zero otherwise.
- $VaR_{0.99}(X) = VaR_{0.99}(Y) = \$0$.
- First of all, The values of VaR for this situation convey a sense of no risk,
- secondly they cannot be used to determine the preference ordering of X and Y.

Example 1

- Instead of comparing X and Y, we consider
- $$U_X(n) = u + n - S_X(n), n \geq 0$$
  where $S_X(n) = \sum_{i=1}^{n} X_i$ and X_i are i.i.d random variables having the same distribution with X; and
- $$U_Y(n) = u + n - S_Y(n), n \geq 0$$
  where $S_Y(n) = \sum_{i=1}^{n} Y_i$ and Y_i are i.i.d random variables having the same distribution as Y..

Example 1

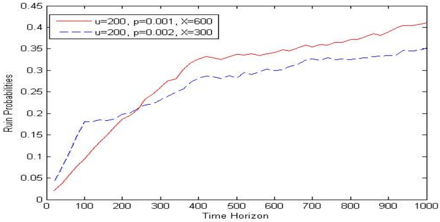

Figure : Ruin probability as risk measure–example 1

Example 1a

- How about TVaR?
- $TVaR_{.99}(X) = \$600$ and $TVaR_{.99}(Y) = \$300$,
- This does indicate that X is more risky than Y according to TVaR at level 0.99. However, because TVaR only considers the tail mean, the change in the values is somewhat too dramatic.

Example 1a

- In addition, consider a risk Z, which, in one time unit, causes no loss with probability .999, however with probability 0.001 may cause a loss whose size can be \$200 or \$1000, each with probability 0.5. Obviously $TVaR_{0.99}(X) = TVaR_{0.99}(Z) = \$600$. However, see figure 2.

Example 1a

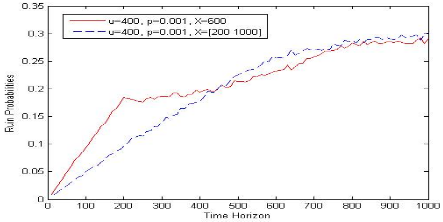

Figure : Ruin probability as risk measure–example 1 continued

Example 2

- In this example, we assume that the net worth of a company follow a jump diffusion process (Merton, 1974; Cummins, 1988)

$$\frac{dU(t)}{U(t)} = \mu dt + \sigma dW(t) + dJ(t), \tag{2}$$

where μ and σ are constant, $W(t)$ is a standard Brownian motion,

Example 2

- $J(t)$ is a compound poisson jump process, which is defined by

$$J(t) = \sum_{i=1}^{N(t)} (Y_i - 1). \tag{3}$$

Here $(Y_i - 1),\ i = 1, 2, \cdots$ are iid random variables represents the percentage change in surplus due to the jumps.

Example 2

- In Merton (1974), it was assume that Y_i, $i = 1, 2, \cdots$ are positive, because the underling process is stock process and must take positive values.
- Here we assume that Y_i, $i = 1, 2, \cdots$ take values in the interval $(-\infty, 1)$ to represent the fact that the jumps are used to model losses and thus must reduce surplus. In addition, the company's net worth can become negative after a jump loss. The process is stopped once it drops below zero.

Example 2

- Because of the diffusion component in this risk process, the simulation of the ruin probability is more complicated. Here we adopted a simulation scheme similar to Becker (2007). In particular, we first simulate the Poisson jumps time and jump sizes and then simulate the minimum of the Geometric Brownian motion process between jumps by applying Corollary 2 of Mcleish (2002).

Example 2

- consider the following two scenarios. The diffusion part in the two scenarios are assumed to be the same, with $\mu = 0.12$, and $\sigma = 0.2$,
- In the first case, $\lambda = 0.01$ and the jump size $Y_i - 1$ are assumed to the negative of an exponentially distributed random variable with mean 1;
- In the second case, $\lambda = 0.1$ and $Y_i - 1$ is the negative of an exponentially distributed random variable with mean 0.1.

Example 2

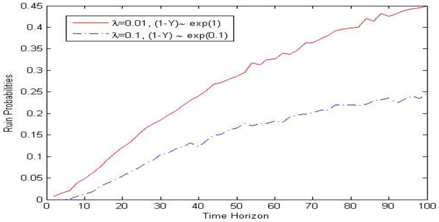

Figure : Ruin probability as risk measure–example 2

Discussions

- VaR and TVaR consider the short term effect of a risk, with which the remote but severe risk cannot be measured and communicated effectively.
- By looking at the long term effect of the risk, long–term risk measures may supplement VaR and its modifications as informative risk measures.

Discussions

- Some might argue that the long–term risk measure should not be relevant to them because their investment horizon is short–term and the risk of a large loss in the short–term is indeed very small.
- The danger is that if one is rewarded with the risk premium for bearing a remote risk for a short time period, it is quite possible for them to repeat the behavior, until a catastrophic event indeed occurs.

Discussions

- A similar situation was commented by Slovic (2002) regarding the risk of smoking "Belief in the near-term safety of smoking may combine in an insidious way with a tendency for young smokers to be uninformed about, or underestimate, the difficulty of stopping smoking."

Discussions

- Using a long–term risk measure necessarily introduces more model and parameter risk
- Consequently, the prediction of ruin probability is by no means accurate.
- However, it is hoped that it can be a useful way to communicate low frequency–high severity risk to decision makers by providing a way to looking at the long–term implication of the short–term risks.

THANK YOU!